**Decision Analytics for Business and Policy: Assignment 5**

Jeff Scanlon

Andrew ID: jscanlo2

The food items are indexed by *i* = 1,…,1007. The nutrients are indexed by *j* = 1,…,10. The food groups are indexed by *k* = 1,…6. Weights on objective function goals are indexed by *w* = 1,…,17. The following parameters and decision variables are introduced.

**Parameters**

r*j* = minimum number of milligrams per day of nutrient j required (or recommended) by WHO

min*k* = minimum proportion (in percent) of food group k required in the WHO diet

max*k* = maximum proportion (in percent) of food group k allowed in the WHO diet

αw = a number between 0 and 1 which represents the weight placed on one goal compared to another

**Decision Variables**

Equations (1) and (2) define the two functions that will constitute the bi-objective function model in Equation (3). Equations (1) and (2) define the total energy and total cost associated with the WHO recommended diet. These equations divide by 100 to account for the fact that the energy (ei) and cost (ci) terms represent coefficients in units per 100 grams of food item *i*, while the decision variable xi is written as a unit of 1 gram of food item *i* per day. Equation (3) makes use of a vector of alpha values to set respective weights on Total Energy and Total Cost, where the weights placed on each of these goals sums to 1. Equation (4) sets the constraint that for every nutrient *j*, the recommended WHO diet must meet the required minimum number of milligrams per day. Equations (5) and (6) set the constraint that for every food group *k* ,the WHO recommended diet must satisfy the minimum proportion of that food group and not exceed the maximum proportion of that food group, respectively.

In python, Equations (3-6) are implemented within a for loop that iterates through the vector of alpha values so that a pareto-optimal frontier can be generated which traces out the optimal solution values when varying weights are applied to the two goals.

**Model 1** *– Weight-Based Approach*

(1)

(2)

(3)

A second model is also created which takes a goal programming approach. Equation (7) shows a new objective function for this model, which is simply to minimize the total energy per child per day in the recommended diet. This objective function can be achieved because an additional constraint is implemented, shown in Equation (8), which limits the total cost per day to $0.50. Thus, Equation (8) becomes a binding constraint, allowing the model to find the lowest possible total energy per day at this defined total cost. Statement (9) indicates that Model 2 is also subject to the previous three constraints listed in Model 1, in addition to the constraint in Equation (8).

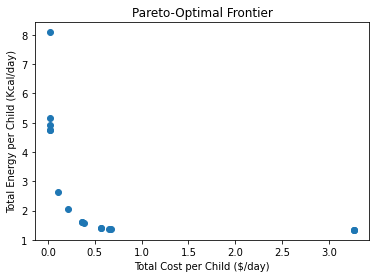
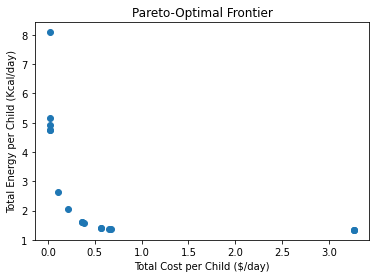
**Model 2** *– Goal Programming*(7)

(8)

(9)

**Computational Results**

After implementation with Gurobi in python, a pareto-optimal frontier is plotted which shows the optimal solution values for each alpha (weight) used in the model. Since WHO would like to minimize both total energy and total cost, the ideal region of the plot is the lower left corner, near the origin (0,0). Thus, the optimal frontier bends toward this region.

**Model 1** *– Weight-Based Approach*

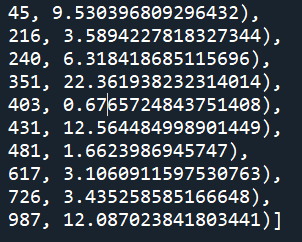
The optimal frontier plot suggests that there are some solutions that are likely not ideal for the WHO to consider. For example, if one places an extremely large weight on either total energy or total cost, the marginal benefits received on that goal are very small. For example, reducing total energy from approximately 1.3 Kcal per day to 1.29 Kcal per day could mean a huge jump in total cost, from $0.50 per day to over $3.00 per day. This trade-off is unlikely to appeal to the WHO. Likewise, a marginal improvement on total cost (a reduction of less than one cent per day), may increase total energy from 5 Kcal per day to 8 Kcal per day. This trade-off is also unlikely to appeal to the WHO. Thus, I have circled a region on the frontier (in red) that I suspect contains the solutions most likely to appeal to WHO, as these solutions have reasonable marginal changes in the two objectives as one moves from one solution to the next.

**Model 2** *– Goal Programming*

If the WHO is able to identify a maximum daily cost that it would like to achieve, then a goal programming approach can be taken to identify the lowest total energy possible that achieves this cost goal. As a case in point, if the WHO states that the total daily cost must not exceed $0.50, we can adjust our model according to the documentation above and receive the following outcome:

Optimal Total Energy: 1.450442861 Kcal/day

This optimal objective value is obtained with the following solution, where, for each pair of numbers, the first number represents food item *i* and the second number represents the decision variable X*i* (the number of grams of that food item per day in the recommended diet).



Conversely, a model could also be devised that identifies the lowest possible total cost at a maximum threshold for total energy per day.